

Fermion number anomaly with the fluffy mirror fermion

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 Quite recently, Grabowska and Kaplan presented a 4-dimensional lattice formulation of chiral gauge theories based on the chiral overlap operator. We study this formulation from the perspective of the fermion number anomaly and possible associated phenomenology. A simple argument shows that the consistency of the formulation implies that the fermion with the opposite chirality to the physical one, the “fluffy mirror fermion” or “fluff”, suffers from the fermion number anomaly in the same magnitude (with the opposite sign) as the physical fermion. This immediately shows that if at least one of the fluff quarks is massless, the formulation provides a simple viable solution to the strong CP problem. Also, if the fluff interacts with gravity essentially in the same way as the physical fermion, the formulation can realize the asymmetric dark matter scenario.

Subject Index B01, B31, B46, B70

1. Introduction

Quite recently [1, 2], Grabowska and Kaplan presented a 4-dimensional lattice formulation of chiral gauge theories based on the chiral overlap operator. They obtained this 4-dimensional formulation by taking the infinite 5th-dimensional extent limit in their 5-dimensional domain-wall formulation in Ref. [3].¹ The procedure is analogous to the one obtained by the overlap formulation [5–8] and the associated overlap Dirac operator [9, 10] from the domain-wall Dirac fermion [11–13]. Although the work of Refs. [14, 15] already paved the way to a manifestly gauge-invariant nonperturbative formulation of chiral gauge theories, the construction of the gauge-invariant fermion measure [14, 15] still remains elusive, except for the abelian case [14] for which the explicit construction is known. See also Refs. [16–18]. Thus, the manifestly gauge-invariant lattice formulation in Refs. [1, 2] appears astonishing in our opinion, because it can avoid the complicated construction of the fermion measure; however, one still has to understand how the locality of the formulation distinguishes the anomaly-free cases from the anomalous cases.

In the present note, we study the formulation in Refs. [1, 2] from the perspective of the fermion number anomaly [19, 20] and possible associated phenomenology. A simple argument shows that the consistency of the formulation implies that the fermion with the opposite chirality to the physical one, the “fluffy mirror fermion” or “fluff”, suffers from the fermion number anomaly in the same magnitude (with the opposite sign) as the physical chiral fermion; the fluff feels the same topological charge as the physical one, contrary to the claim made in Refs. [1, 2]. This must be so simply because the number symmetry that rotates both the physical and the fluff fermions is an exact symmetry of the formulation. Assuming the validity of the formulation, especially the restoration of the locality for anomaly-free chiral gauge theories, we accept the existence of the fluff fermions positively and discuss possible phenomenological implications. The above property of the fermion number anomaly immediately shows that if at least one of the fluff quarks is massless, the formulation provides a simple viable solution to the strong CP problem; this is the massless up-quark solution in the mirror or fluff sector. Also, if the fluff interacts with gravity essentially in the same way as the physical fermions, its fermion number violation can provide an understanding of the coincidence between the baryon and the dark matter abundances, realizing the asymmetric dark matter scenario [21, 22].

2. Fermion number anomaly with the chiral overlap operator

In the formulation of Refs. [1, 2], the partition function of the (left-handed) Weyl fermion is given by²

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left(-a^4 \sum_x \bar{\psi}(x) \mathcal{D}_\chi \psi(x) \right), \quad (2.1)$$

where a denotes the lattice spacing and \mathcal{D}_χ is the chiral overlap operator defined by

$$a\mathcal{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} (1 - \epsilon) \right]. \quad (2.2)$$

¹ For a closely related 6-dimensional domain-wall formulation, see Ref. [4].

² In what follows, the lattice gauge field is treated as a nondynamical background, because the integration over the gauge field is irrelevant in the following discussion.

Here, ϵ is the sign function

$$\epsilon = \epsilon(H_w) = \frac{H_w}{\sqrt{H_w^2}} \quad (2.3)$$

of the hermitian Wilson Dirac operator

$$H_w = \gamma_5 \left[\frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - \frac{1}{2} a \nabla_\mu \nabla_\mu^* - m \right], \quad (2.4)$$

where ∇_μ and ∇_μ^* are forward and backward gauge-covariant lattice derivatives, respectively; H_w is a function of the gauge field A . The gauge field contained in ϵ_\star , denoted by A_\star , is on the other hand defined by a one-parameter deformation (the gradient flow) of the original 4-dimensional gauge field A [1, 2]; i.e.,

$$\epsilon = \epsilon(H_w[A]), \quad \epsilon_\star = \epsilon(H_w[A_\star]). \quad (2.5)$$

The crucial point in the formulation is that the fermion integration variables in Eq. (2.1) are 4-component Dirac fields without any chirality constraint. This is quite in contrast to the formulation of Refs. [14, 15] and for this reason one can avoid the construction of the nontrivial fermion measure. The naive continuum limit of Eq. (2.2) is [1, 2]

$$\mathcal{D}_\chi \xrightarrow{a \rightarrow 0} \frac{1}{am} \begin{pmatrix} 0 & \sigma_\mu D_\mu(A) \\ \bar{\sigma}_\mu D_\mu(A_\star) & 0 \end{pmatrix}. \quad (2.6)$$

Therefore, in this naive continuum limit, the left-handed fermion interacts with A and the right-handed one interacts with A_\star . If the gauge field A_\star is basically trivial, then the system may be regarded as the left-handed Weyl fermion interacting with the gauge field [23]. In Refs. [1, 2], A_\star is defined by the gradient flow [24–27] along the 5th-dimensional direction of the domain-wall formulation in Ref. [3].

Now, using $\epsilon^2 = \epsilon_\star^2 = 1$,

$$\epsilon + \epsilon_\star = \epsilon(1 + \epsilon\epsilon_\star) \quad (2.7)$$

and one can confirm that

$$\left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} (1 - \epsilon) \right]^2 = 1 \quad (2.8)$$

and, consequently, \mathcal{D}_χ satisfies the Ginsparg–Wilson relation [28]

$$\gamma_5 \mathcal{D}_\chi + \mathcal{D}_\chi \gamma_5 = a \mathcal{D}_\chi \gamma_5 \mathcal{D}_\chi. \quad (2.9)$$

This Ginsparg–Wilson relation allows one to define a modified γ_5 [29, 30],

$$\hat{\gamma}_5 \equiv \gamma_5 (1 - a \mathcal{D}_\chi), \quad (2.10)$$

which satisfies³

$$(\hat{\gamma}_5)^2 = 1, \quad \mathcal{D}_\chi \hat{\gamma}_5 = -\gamma_5 \mathcal{D}_\chi. \quad (2.11)$$

Thus, we can define projection operators,

$$\hat{P}_\pm \equiv \frac{1}{2} (1 \pm \hat{\gamma}_5), \quad P_\pm \equiv \frac{1}{2} (1 \pm \gamma_5), \quad (2.12)$$

³ Note that in general $(\hat{\gamma}_5)^\dagger \neq \hat{\gamma}_5$. We would like to thank Okuto Morikawa for pointing this out to us.

and chiral components of the fermion by

$$\hat{P}_- \psi_L(x) = \psi_L(x), \quad \bar{\psi}_L(x) P_+ = \bar{\psi}_L(x), \quad (2.13)$$

$$\hat{P}_+ \psi_R(x) = \psi_R(x), \quad \bar{\psi}_R(x) P_- = \bar{\psi}_R(x). \quad (2.14)$$

Then, thanks to the last relation of Eq. (2.11), the action is completely decomposed into the left and right fermion components as

$$a^4 \sum_x \bar{\psi}(x) \mathcal{D}_\chi \psi(x) = a^4 \sum_x [\bar{\psi}_L(x) \mathcal{D}_\chi \psi_L(x) + \bar{\psi}_R(x) \mathcal{D}_\chi \psi_R(x)]. \quad (2.15)$$

Let us now introduce the fermion number transformations. For the left-handed “physical” fermion, the fermion number $U(1)$ transformation is defined by

$$\psi_L(x) \rightarrow e^{i\theta} \psi_L(x), \quad \bar{\psi}_L(x) \rightarrow e^{-i\theta} \bar{\psi}_L(x), \quad (2.16)$$

and $\psi_R(x)$ and $\bar{\psi}_R(x)$ are kept intact. On the other hand, for the right-handed “invisible” or “fluff” fermion,

$$\psi_R(x) \rightarrow e^{i\theta} \psi_R(x), \quad \bar{\psi}_R(x) \rightarrow e^{-i\theta} \bar{\psi}_R(x), \quad (2.17)$$

and $\psi_L(x)$ and $\bar{\psi}_L(x)$ are kept intact. We note that the combination of these two, which defines the sum of fermion numbers of both chiralities,

$$\psi_L(x) \rightarrow e^{i\theta} \psi_L(x), \quad \bar{\psi}_L(x) \rightarrow e^{-i\theta} \bar{\psi}_L(x), \quad (2.18)$$

$$\psi_R(x) \rightarrow e^{i\theta} \psi_R(x), \quad \bar{\psi}_R(x) \rightarrow e^{-i\theta} \bar{\psi}_R(x), \quad (2.19)$$

or, equivalently,

$$\psi(x) \rightarrow e^{i\theta} \psi(x), \quad \bar{\psi}(x) \rightarrow e^{-i\theta} \bar{\psi}(x), \quad (2.20)$$

leaves the action and the fermion measure in Eq. (2.1) invariant. Therefore, this symmetry is exact and anomaly-free.

The fermion number of the left-handed physical fermion is anomalous. The action (2.15) is invariant under Eq. (2.16), but the measure is not because of the chirality projection (2.13) which nontrivially depends on the gauge field. Considering the transformation (2.16) with the localized parameter $\theta \rightarrow \theta(x)$, we have the anomalous conservation law of the fermion number current of the left-handed fermion,

$$\begin{aligned} \langle \partial_\mu j_{L\mu}(x) \rangle &= \text{tr} [\hat{P}_-(x, x) - P_+ \delta_{x,x}] \\ &= \frac{1}{2} \text{tr} [\gamma_5 a \mathcal{D}_\chi(x, x)], \end{aligned} \quad (2.21)$$

where tr stands for the trace over the spinor and gauge indices only and $\hat{P}_-(x, y)$, e.g., denotes the kernel of \hat{P}_- in the position space; we have used $\text{tr} \gamma_5 = 0$ in the second equality. Similarly, the fermion number current of the right-handed fermion does not conserve:

$$\begin{aligned} \langle \partial_\mu j_{R\mu}(x) \rangle &= \text{tr} [\hat{P}_+(x, x) - P_- \delta_{x,x}] \\ &= -\frac{1}{2} \text{tr} [\gamma_5 a \mathcal{D}_\chi(x, x)]. \end{aligned} \quad (2.22)$$

The sum of these two conserves

$$\langle \partial_\mu [j_{L\mu}(x) + j_{R\mu}(x)] \rangle = 0 \quad (2.23)$$

as should be from the invariance of the system under Eq. (2.20). Already Eqs. (2.21) and (2.22) show that the left-handed physical fermion and its mirror partner, the right-handed fluff, suffer from the fermion number anomaly in the same magnitude (with the

opposite sign), no matter how A and A_\star are related. This is contrary to the claims of Refs. [1, 2] that the flowed field A_\star , which couples to the right-handed fermion, is pure-gauge and thus carries no topological information about the original gauge field A .

That A and A_\star must carry the same topological information can also be seen from the nonconservation of the fermion number charge, which is obtained by integrating Eq. (2.21) over 4-dimensional spacetime. Using Eq. (2.2), we have

$$\begin{aligned}\langle Q_L(x_0 = +\infty) \rangle - \langle Q_L(x_0 = -\infty) \rangle &= \frac{1}{2} \text{Tr} \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} (1 - \epsilon) \right] \\ &= \frac{1}{2} \text{Tr} \left[1 - (1 - \epsilon)(1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} \right] \\ &= \frac{1}{2} \text{Tr} \left[(\epsilon + \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} \right] \\ &= \frac{1}{2} \text{Tr} \epsilon.\end{aligned}\tag{2.24}$$

In this expression, $\text{Tr} \equiv a^4 \sum_x \text{tr}$ and we have used the cyclic property of Tr in the first equality. In the last equality, we have noted Eq. (2.7). However, we can equally use $\epsilon + \epsilon_\star = (1 + \epsilon\epsilon_\star)\epsilon_\star$ to conclude

$$\langle Q_L(x_0 = +\infty) \rangle - \langle Q_L(x_0 = -\infty) \rangle = \frac{1}{2} \text{Tr} \epsilon_\star.\tag{2.25}$$

Thus, for the expression (2.2) to be meaningful, $\text{Tr} \epsilon_\star = \text{Tr} \epsilon$. In particular, A_\star cannot be pure gauge, for which we can see that $\text{Tr} \epsilon_\star = 0$.

The above argument implicitly assumes that $\text{Tr} \epsilon$ can be nonzero. This is actually the case as can be seen in the classical continuum limit [31–34],

$$\begin{aligned}\frac{1}{2} \text{tr} \epsilon(x, x) &= \frac{1}{2} \text{tr} \frac{H_w}{\sqrt{H_w^2}}(x, x) \\ &\xrightarrow{a \rightarrow 0} -\frac{1}{32\pi^2} I(am, 1) \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu}(x) F_{\rho\sigma}(x)]\end{aligned}\tag{2.26}$$

for a smooth background gauge field, where the function $I(am, r)$ is given by

$$I(am, r) = \theta(am/r) - 4\theta(am/r - 2) + 6\theta(am/r - 4) - 4\theta(am/r - 6) + \theta(am/r - 8)\tag{2.27}$$

from the step function $\theta(x)$. For $0 < am < 2$ for which the free lattice Dirac operator possesses only a single zero in the Brillouin zone, $I(am, 1) = 1$ and Eq. (2.26) reproduces the correct fermion number anomaly in the classical continuum limit.⁴ Thus, $\text{Tr} \epsilon = a^4 \sum_x \text{tr} \epsilon(x, x)$ can actually be nonzero for a topologically nontrivial background.

It is of interest to obtain the classical continuum limit of the right-hand side of Eq. (2.21) which depends on both A and A_\star :

$$\mathcal{A}_L(x) \equiv \frac{1}{2} \text{tr} \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\epsilon_\star + 1} (1 - \epsilon) \right] (x, x).\tag{2.28}$$

We could not complete the explicit perturbative calculation, but a general argument suggests its form in the classical continuum limit as follows. First, since ϵ and ϵ_\star are hermitian

⁴ We note that the topological charge is given by $Q_{\text{top.}} = \int d^4x (1/64\pi^2) \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x)$ (for the instanton, $Q_{\text{top.}} = 1$) and thus in the continuum limit, $\langle Q_L(x_0 = +\infty) \rangle - \langle Q_L(x_0 = -\infty) \rangle = 2N_f T(R) Q_{\text{top.}}$, where N_f is the number of flavors and $\text{tr}(T^a T^b) = -T(R) \delta^{ab}$.

operators, $\mathcal{A}_L(x)[A_\star, A] = \mathcal{A}_L(x)[A, A_\star]^*$. Next, from Eqs. (2.24), (2.25), and (2.26),

$$\mathcal{A}_L(x) \xrightarrow{a \rightarrow 0} -\frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu}(x) F_{\rho\sigma}(x)] + \partial_\mu X_\mu(x), \quad (2.29)$$

$$\mathcal{A}_L(x) \xrightarrow{a \rightarrow 0} -\frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\star\mu\nu}(x) F_{\star\rho\sigma}(x)] + \partial_\mu Y_\mu(x), \quad (2.30)$$

where $X_\mu(x)$ and $Y_\mu(x)$ are gauge-invariant vectors. From these properties, we conjecture that the classical continuum limit of $\mathcal{A}_L(x)$ is given by

$$\mathcal{A}_L(x) \xrightarrow{a \rightarrow 0} -\frac{1}{64\pi^2} \{ \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu}(x) F_{\rho\sigma}(x)] + \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\star\mu\nu}(x) F_{\star\rho\sigma}(x)] \}. \quad (2.31)$$

This is obviously consistent with the property $\mathcal{A}_L(x)[A_\star, A] = \mathcal{A}_L(x)[A, A_\star]^*$. Also noting the identity,

$$\begin{aligned} & \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\star\mu\nu}(x) F_{\star\rho\sigma}(x)] \\ &= \epsilon_{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu}(x) F_{\rho\sigma}(x)] + \partial_\mu \int_0^\infty dt \, 4\epsilon_{\mu\nu\rho\sigma} \text{tr} \left[\frac{\partial}{\partial t} A_\nu(t, x) F_{\rho\sigma}(t, x) \right], \end{aligned} \quad (2.32)$$

where $A_\mu(t, x)$ is a one-parameter family of the gauge fields that connects A and A_\star ,

$$A_\mu(t=0, x) = A_\mu(x), \quad A_\mu(t=\infty, x) = A_{\star\mu}(x), \quad (2.33)$$

Eqs. (2.29) and (2.30) are satisfied if $\frac{\partial}{\partial t} A_\nu(t, x)$ is gauge covariant. The simplest possibility for this would be the gradient flow

$$\frac{\partial}{\partial t} A_\mu(t, x) = D_\nu F_{\nu\mu}(t, x). \quad (2.34)$$

3. Phenomenological/cosmological implications

In this section, we discuss possible phenomenological/cosmological implications of the lattice formulation in Refs. [1, 2]. We have observed that, in this lattice formulation of the chiral gauge theory, the fermion number violation caused by the chiral gauge interaction for the “physical” or “visible” fermion necessarily accompanies the opposite fermion number violation for the opposite chirality “invisible” fermion, the “fluff”. If the standard model (SM) is nonperturbatively defined by the present lattice formulation (“fluffy SM”), all the SM fermions accompany their mirror partners. This fact immediately implies that if at least one of the fluff quarks is massless, the θ angle for the strong interaction can be rotated away by the chiral rotation of the fluff. This is simply the massless up-quark solution to the strong CP problem in the fluff or mirror sector. Note that the structure of Yukawa couplings in the fluff sector (see below) can be quite different from the physical sector.

The properties of the formulation also possibly have implications for the baryogenesis and the dark matter problem. The $B + L$ charge violation that may be caused by the instanton or sphaleron effect in the electroweak sector always accompanies a $B + L$ violation with the same magnitude (with an opposite sign) for the fluff; the sum $B + L + (B + L)_{\text{fluff}}$ is conserved. The $B - L$ charges in the visible sector and those in the fluff sector are, on the other hand, separately conserved. These properties provide a realization of the asymmetric dark matter scenario [21, 22, 35–39] as follows.

First, we assume that the left-handed physical (or anti-right-handed) fermions and the right-handed fluff interact with gravity essentially in the same way, i.e., the fermion is coupled

to the gravity as an ordinary Dirac fermion, as naturally suggested from the action (2.15) in flat spacetime.⁵

Suppose further that the fluff is thermalized after the inflation, e.g., via a coupling with the inflaton. Then it will contribute to the relativistic degrees of freedom other than the photon, N_{eff} , at the epoch of the recombination. N_{eff} is constrained from the observation of the cosmic microwave background as $N_{\text{eff}} < 3.13 \pm 0.32$ [40]. Therefore, it is plausible that the fluff acquires mass and annihilates into the visible particles (or to a dark radiation if we allow $N_{\text{eff}} = 4$) via interactions other than those from the SM gauge group. For example, we can couple a Higgs field to the fluff to make them heavy,⁶

$$y\bar{u}_L H q_R + \text{h.c.}, \quad (3.1)$$

where u_L denotes the “fluff up quark” and q_R the “fluff quark doublet”. To make the annihilation process efficient, we may further introduce a light scalar field that couples to the fluffs with nonhierarchical Yukawa couplings; another possibility is to introduce a light $B - L$ gauge boson that couples to the fluffs by the minimal coupling and to the physical fermions through the gradient flow.

Once the baryon or lepton number is produced in either side (e.g., as in electroweak baryogenesis [41–44] or leptogenesis [45]) when the sphaleron process is at work, it will be redistributed between the two sectors [36, 37].⁷ Thus the two sectors obtain a similar size of baryon and lepton numbers. Assuming the SM gauge group, N_g generations and N_h Higgs doublets, the conserved charges are Cartan generators of $SU(3)_c$ and $SU(2)_L$, the $U(1)_Y$ hypercharge, the difference between the baryon and lepton numbers in the physical sector $B - L$, that in the fluff sector $(B - L)_{\text{fluff}}$,⁸ and the sum of the baryon and lepton numbers $B + L + (B + L)_{\text{fluff}}$. Then the standard argument [46, 47] on the basis of the relativistic free particle approximation yields, at the chemical equilibrium,

$$n_B = \frac{21}{52}n_{B-L} + \frac{1}{4}n_{B+L+(B+L)_{\text{fluff}}} + \frac{5}{52}n_{(B-L)_{\text{fluff}}}, \quad (3.2)$$

$$n_L = -\frac{31}{52}n_{B-L} + \frac{1}{4}n_{B+L+(B+L)_{\text{fluff}}} + \frac{5}{52}n_{(B-L)_{\text{fluff}}}, \quad (3.3)$$

$$n_{B_{\text{fluff}}} = \frac{5}{52}n_{B-L} + \frac{1}{4}n_{B+L+(B+L)_{\text{fluff}}} + \frac{21}{52}n_{(B-L)_{\text{fluff}}}, \quad (3.4)$$

$$n_{L_{\text{fluff}}} = \frac{5}{52}n_{B-L} + \frac{1}{4}n_{B+L+(B+L)_{\text{fluff}}} - \frac{31}{52}n_{(B-L)_{\text{fluff}}}. \quad (3.5)$$

For example, for the standard leptogenesis for which initially $n_L \neq 0$ and $n_B = n_{B_{\text{fluff}}} = n_{L_{\text{fluff}}} = 0$, we have $n_{B_{\text{fluff}}} = n_{L_{\text{fluff}}} = -n_B$ at the equilibrium. It is interesting that in the present formulation, Eqs. (3.2)–(3.5) depend on neither N_g nor N_h . Also, note that $B + L$ produced in the early universe is not washed out and split into the two sectors. $n_{B_{\text{fluff}}}$

⁵ Such a left–right symmetric gravitational coupling will imply the equality (with the opposite sign) of the gravitational contributions to the fermion number anomaly between the physical fermion and the fluff. It will also be interesting to investigate the possible implications of the gravitational interaction between the physical and fluff sectors.

⁶ To make the “fluff neutrino” massive, we may turn off the seesaw mechanism by eliminating the Majorana neutrino mass if $B - L$ is global, while we tune the neutrino Yukawa couplings if $B - L$ is gauged.

⁷ This scenario is essentially the fluff version of Ref. [37] with $X = (B + L)_{\text{fluff}}$.

⁸ Here we consider the case in which the $B - L$ breaking effects are decoupled.

and $n_{L_{\text{fluff}}}$ are fixed after the universe is cooled down below the temperature at which the sphaleron process is effective. Then if pair-annihilation processes are efficient enough, the symmetric part of the fluff sector disappears. The remaining fluff fermion (or minus anti-fermion) numbers are proportional to $n_{B_{\text{fluff}}}$ and $n_{L_{\text{fluff}}}$ and provide the observed dark matter abundance $\Omega_{\text{DM}} \simeq 5\Omega_B$ if the average of fluff fermion masses per baryon or lepton number is $O(5)$ GeV.^{9 10} It is interesting to observe that the SM $SU(3)$ gauge interaction does not contribute to the renormalization group running of the Yukawa couplings of the fluff quarks. This would reduce the ratio of the “fluff top” Yukawa coupling and the physical top Yukawa coupling even if they are equal at the cutoff scale.

Another interesting observation is that the visible quarks do not contribute to the neutron electric dipole moment (EDM) after the strong CP phase is rotated away to the fluff quarks. Thus the stringent bound on the strong CP-violating phase could be ameliorated even if all the fluff quarks are massive. It is tempting to imagine the phase participates in the baryo/leptogenesis. Actually, the fluff quarks couple to the neutron EDM via the vacuum polarization of the physical Higgs or the $B - L$ gauge field. It is at least one-loop suppressed. In addition, the diagram is suppressed by the two small visible Yukawa couplings or flowed $B - L$ couplings. Also, the CP-violating fluff-Higgs coupling is suppressed by the smallest fluff Yukawa coupling according to the standard calculation of the EDM [50].

The above fluff dark matter interacts with the SM particle via the Higgs portal [51–58] or a $B - L$ gauge field [59–63]. They can be explored or constrained by direct/indirect or collider dark matter searches. In the former case, they also contribute to the invisible decay width of the SM Higgs boson. In the latter case, the $B - L$ gauge boson will be searched by the LHC and future colliders. The dark matter is multicomponent in the minimal setup and its precise nature depends on the hierarchy of the fluff Yukawa couplings.

We have briefly sketched phenomenological/cosmological implications of the formulation. The construction of working phenomenological models and their detailed numerical analyses are beyond the scope of this paper; we leave them to the future work. It should be noted that the present lattice formulation, in principle, allows one to carry out nonperturbative field theoretical analyses of, e.g., the electroweak baryogenesis in the fluffy SM, from first principles.

Note added

An explicit calculation [64] of the classical continuum limit of the fermion number anomaly (2.28) reveals that the conjectured form (2.31) is wrong; the correct expression turns out to be much more complicated as shown in Ref. [64].

⁹ If masses of the fluff fermions are provided by the Yukawa coupling to the SM Higgs field, the Yukawa couplings to the fluff fermions are constrained by the upper bound on the Higgs invisible branching ratio at LHC. The fluff mass of $O(1)$ GeV is barely consistent with the present bound [48, 49] and the case of the averaged fluff fermion mass of $O(5)$ GeV might be excluded. This number however can be reduced within the present scenario by assuming that $n_B \neq 0$ and $n_L \neq 0$ for the primordial densities.

¹⁰ In the minimal setting, the flavor is preserved in the “invisible” sector since the weak interaction is effectively suppressed by the gradient flow.

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